ABSTRACT

The present thesis entitled “A study of pseudo-differential operators involving fractional Fourier transform” embodies the results of research carried out by author. Pseudo-differential operators are generalization of partial differential operators and have been defined on various function spaces and distributional spaces by using many integral transforms. In fact, these operators played an important role to study the elliptic partial differential equations. These operators are also useful to consider many problems such as existence and uniqueness of boundary value problems in linear partial differential equations, regularity of elliptic partial differential equations and also yield many significant results in non linear partial differential equations.

Fractional Fourier transform has been frequently used to study the pseudo-differential operators and wavelet transform in the recent years.

My main purpose of the present thesis is to focus the development of theory of pseudo-differential operators, wavelet transform on $W$-type spaces and asymptotic series of general symbol of pseudo-differential operator on Schwartz space $S(\mathbb{R})$ with help of fractional Fourier transform technique.

This thesis consists of five chapters. Chapter 1, gives the brief introduction about various definitions and properties of $W$-type spaces, Fourier transform, fractional Fourier transform, pseudo-differential operators, wavelet transform, Schwartz space, tempered distribution. The relation between Fourier transform and fractional Fourier transform is also given.

In Chapter 2, characterization of $W$-type spaces is investigated and various mapping properties, an integral representation of pseudo-differential operators are obtained by using fractional Fourier transform.

In Chapter 3, characterization of $W^p$-type spaces is defined and the relations between $W$ and $W^p$ type spaces are found by exploiting fractional Fourier transform. The uniqueness class of Cauchy problem is also studied by using the aforesaid transform.
In Chapter 4, \( n \)-dimensional continuous fractional wavelet transform (CFrWT) is introduced and its properties discussed. Various mapping properties of fractional wavelet transform on Gel’fand and Shilov spaces of type \( W_M(\mathbb{R}^n) \), \( W^\Omega(\mathbb{C}^n) \) and \( W^{\Omega\ast}_M(\mathbb{C}^n) \) are given.

In Chapter 5, the concept of generalized Sobolev space \( \mathcal{G}^{s,p}_\alpha(\mathbb{R}) \), involving fractional Fourier transform is defined. Some bounded estimates of pseudo-differential operator associated with \( L^p \)-norm for \( 1 \leq p < \infty \) are discussed and an asymptotic series of general symbol of pseudo-differential operator is also obtained by using the theory of fractional Fourier transform.