SOME COSMOLOGICAL MODELS OF EARLY AND LATE UNIVERSE

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By

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Abstract

We study the effects of introducing modifications to General Relativity (GR) at large scales as an alternative to exotic forms of matter required to respond the observed cosmic acceleration. We survey the effects on cosmological tests of f(R,T) gravity, C-field theory, Brans-Dicke theory, Lyra Geometry, Variable Equation of State (EoS), Holographic Dark Energy. We find that, in addition to the changes to the background expansion history of the Universe, these modifications have substantial impact on structure formation and its observable predictions.

In the construction of a cosmological model, the assumption of homogeneity and isotropy of the Universe are motivated by the cosmological principle and mathematical tractability of the resulting FRW models. So these symmetries can only be approximate. There are theoretical arguments and recent experimental data regarding cosmic background radiation anisotropies which support the existence of an anisotropic phase that approaches an isotropic one. These observations led us to consider more general anisotropic cosmologies, while retaining the assumption of large scale spatial homogeneity. Wainright and Ellis (1997) have discussed the spatially homogeneous and anisotropic cosmological models which provide a richer structure, both geometrically and physically, than the FRW model and play a significant role in the description of early Universe.

The spatially homogeneous cosmological models allow extension of cosmological investigations to distorting and rotating Universes, giving estimates of effects of anisotropy on primordial element production and on the measured CMBR spectrum anisotropy. Apart from observational reasons, there are various theoretical considerations that have motivated the study of anisotropic cosmologies.

In this thesis we investigate some current topics in theoretical cosmology, covering issues which are important at early as well as late times of the cosmological evolution.

The thesis entitled “Some Cosmological Models of Early and Late Universe” has been divided into nine chapters. The first chapter is of introductory nature. However, a brief introduction is given in the beginning of each chapter also.

First chapter contains a brief review of cosmological models, FRW model,
Inflationary Cosmology, Time-varying Cosmological Term ($\Lambda$) and Gravitational constant ($G$), Creation Field Theory, f(R,T) Gravity Theory, Brans Dicke Theory, Lyra Geometry, Particle Creation and Bulk Viscosity, Dark Energy and Holographic Dark Energy, relevant to the remaining chapters of the thesis is presented. However, each chapter contains a brief introduction. At the end, the motivation of the present work has also been given and scope of future work is outlined.

**Second chapter** presents the new class of cosmological models of the early Universe in $f(R,T)$ modified theories of gravity. The exact solutions to the corresponding field equations are obtained in quadrature form. The cosmological parameters have been discussed in detail and it is also shown that the solutions tend asymptotically to isotropic Friedmann-Robertson-Walker cosmological model. We have also discussed the well-known astrophysical parameters, namely the Hubble parameter $H(z)$, luminosity distance ($d_L$) and distance modulus $\mu(z)$ with redshift.

**Third chapter** deals the general class of Bianchi cosmological models in $f(R,T)$ modified theories of gravity with $\Lambda(T)$. This chapter deals with $f(R,T)$ modified theories of gravity proposed by Harko et al (2011), where the gravitational Lagrangian is given by an arbitrary function of Ricci scalar $R$ and the trace of the stress-energy tensor $T$, for a specific choice of $f(R,T) = f_1(R) + f_2(T)$. The exact solutions to the corresponding field equations are obtained in quadrature form. We have discussed three types of solutions of the average scale factor for the general class of Bianchi cosmological models by using a special law for deceleration parameter which is linear in time with a negative slope. The solutions to the Einstein field equations are obtained for three different physically viable cosmologies.

**Fourth chapter** deals with the solutions of Einstein’s equations with cosmological constant ($\Lambda$) in the presence of a creation field for general class of anisotropic cosmological models. We have obtained the cosmological solutions for two different scenarios of average scale factor. In the first case, we have discussed three different types of physically viable cosmological solutions of average scale factor for the general class of Bianchi cosmological models by using a spe-
cial law for deceleration parameter which is linear in time with a negative slope. In the second case, we have discussed three different forms of cosmological solutions by using the average scale factor in three different scenarios like Intermediate scenario, Logamediate scenario and Emergent scenario. We have also discussed the all energy conditions in each cases.

Fifth chapter is based on the general class of Bianchi cosmological models with viscous fluid and particle creation in Brans-Dicke theory. The present chapter deals with the general class of Bianchi cosmological models with bulk viscosity and particle creation described by full causal thermodynamics in Brans-Dicke theory. We have discussed three types of solutions of the average scale factor for the general class of Bianchi cosmological models by using a special law for deceleration parameter which is linear in time with a negative slope. The exact solutions to the corresponding field equations are obtained in quadrature form. The solutions to the Einstein field equations are obtained for three different physically viable cosmologies.

Sixth chapter presents a new class of cosmological models of the early Universe filled with perfect fluid in Lyra’s geometry. We have obtained two classes of exact solutions of the field equations in Lyra’s geometry with a time-dependent displacement vector. The exact solutions to the corresponding field equations are obtained in quadrature form. The cosmological parameters have been discussed in detail and it is also shown that the solutions tend asymptotically to isotropic Friedmann-Robertson-Walker cosmological model. We have also calculated and discussed the jerk parameter for each case.

Seventh chapter is based on the general class of Bianchi cosmological models with dark energy in the form of standard and modified chaplygin gas with variable \( \Lambda \) and \( G \) and bulk viscosity. We have discussed three types of average scale factor by using a special law for deceleration parameter which is linear in time with negative slope. The exact solutions to the corresponding field equations are obtained. We have obtained the bulk viscosity \( (\xi) \), cosmological constant \( (\Lambda) \), gravitational parameter \( (G) \) and deceleration parameter \( (q) \) for different equations of state. It is also shown that the model describes an accelerating Universe for large value
of time $t$, where in the effective negative pressure induced by Chaplygin gas and bulk viscous pressure is driving the acceleration.

**Eighth chapter** deals with the general class of Bianchi cosmological models with varying equation of state (EoS) parameter. We have discussed three different types of physically viable cosmological solutions of average scale factor by using a special law for deceleration parameter which is linear in time with a negative slope. The exact solutions to the corresponding field equations are obtained for three different physically viable cosmologies. The EoS parameter for deceleration parameter as well as dark energy is found to be a time varying function. We have, using the latest observational data, drawn a qualitative picture of the evolution of the Universe. In our model, the equation of state parameter of dark energy is obtained as time varying and it is evolving with negative sign which is consistent with recent observation. We also shown that, at the early stage, the equation of state (EoS) parameter $\gamma$ is positive i.e. the Universe was matter dominated but at large time, the Universe evolving with negative values i.e. the present epoch. All physical parameters are calculated and discussed in each physically viable cosmological model.

**Ninth chapter** presents the general class of Bianchi Cosmological models with holographic dark energy component. We have discussed three types of solutions of the average scale factor for the general class of Bianchi cosmological models by using a special law for deceleration parameter which is linear in time with a negative slope. The exact solutions to the corresponding field equations are obtained. All the physical parameters are calculated and discussed in each physically viable cosmological model. For large time (i.e. when $t \to \infty$) the models tend asymptotically to an isotropic Friedmann-Robertson-Walker cosmological model. Quintessence scalar field and quintessence potential are also obtained for three different scenario of scale factor. We have also compared our models with SNI$_a$ data graphically. It has been found that all the models are in fair agreement with the observational results.